Exercise 5

Differentiate.

$$y = \frac{x}{e^x}$$

Solution

Use the quotient rule to differentiate y.

$$y' = \frac{d}{dx}\left(\frac{x}{e^x}\right)$$

$$= \frac{\left[\frac{d}{dx}(x)\right](e^x) - \left[\frac{d}{dx}(e^x)\right](x)}{(e^x)^2}$$

$$= \frac{(1)(e^x) - (e^x)(x)}{(e^x)^2}$$

$$= \frac{e^x(1-x)}{(e^x)^2}$$

$$= \frac{1-x}{e^x}$$

Alternatively, the product rule can be used.

$$y' = \frac{d}{dx}(xe^{-x})$$

$$= \left[\frac{d}{dx}(x)\right](e^{-x}) + (x)\left[\frac{d}{dx}(e^{-x})\right]$$

$$= (1)(e^{-x}) + (x)(-e^{-x})$$

$$= e^{-x}(1-x)$$

$$= \frac{1-x}{e^x}$$